Higher-Order Thinking via Mathematical Problem Posing Tasks among Engineering Students

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Abstract: This study characterizes engineering students' higher-order thinking (HOT) skills through two problem posing strategies, namely "What..., if not?" and "Modifying Given". Problem posing tasks were designed based on integral concepts from calculus textbook problems. The data were collected through students' responses to test and semi-structured interview. Twenty-six participants in the test session were selected among moderate and high achievers first year engineering students. After the test, 5 of the students were purposefully selected for the semi-structured interview. The results reveals that the HOT skills via problem posing tasks can be characterized into "Interpreting the problem condition and demand in term of mathematics communication", "Manipulating information for constructing new problems in flexibility method", "Analysing the constructed problem regard to solvable or unsolvable", "Create new and different problem which are solvable", "Conclude a significant pattern or structure", and "Finding the differences and similarities between two parts of tasks strategies". The results confirm that problem posing tasks posses all criteria of a practical task for enhancing HOT skills among engineering students.

Introduction

A number of definitions of higher-order thinking have been proposed. According to Newman (1991), higher order thinking is defined broadly as challenge and expanded use of the mind when a person must interpret, analyze, or manipulate information, because a question needs to be answered. Newman asserted that critical, logical, reflective, creative thinking and metacognitive skills can be subsumed under a more general distinction between higher order and lower order thinking. He indicated that lower order thinking represents routine, mechanistic application, and limited use of the mind. In the other word, it involves repetitive routines such as listing information previously memorized, inserting numbers into previously learned formulae, or applying the rules for footnote format in a research paper. In the contrast, Higher-Order Thinking, or HOT for short, is thinking on a higher level than memorizing facts, so that it requires the tasks to be understood, connected to each other, categorized, manipulated, put together in new or novel ways, and applied as new solutions to new problems (Thomas, Thorne, and Small , 2000). Despite of the importance of HOT's achievements in an effectiveness education, teaching and learning method and material in mathematics classrooms, have reminded this phenomena under shadow low-order thinking.

Over the decades, researchers have found that HOT can be taught, nurtured, and developed among learners. To achieve HOT, students should be involved in understanding and transformation of knowledge which are described as the ultimate goals of learning (Bigge, 1976). In this regard, various communications have presented criteria of valid tasks that can foster educators' HOT; however, the most offered activities are inapplicable in mathematics classrooms which are limited to textbook content and time. Shu Mei and Yan (2005) noted that HOT rarely exists among mathematics classrooms in Singapore due to mathematics teaching and learning environments which are teacher-centred and involve absolutely, routine procedural skills and basic concepts. Certainly, these pedagogy achievements are learners who have little understanding of the basic concepts of pre-calculus, and even the better students, only excel in a procedural way of thinking. Furthermore, researchers (Engelbrecht, Bergsten, and Kågesten, 2009) revealed that this condition can also exist in the universities mathematics classrooms, because most tasks and examination tests are considered procedural in character, and are more formal in the concepts. Therefore, educational experts have taken efforts to design appropriate teaching-learning materials and activate which can foster HOT among educators.

Weiss (2003) classified problems which can foster HOT into: Collaborative, Authentic, Ill- Structured problems, and Challenging problems. According to this view, the most important criteria for promoting HOT can be followed as:

- Students should be involved in the transformation of knowledge and understanding.
- Teacher should create a communicating environment for students' effective interaction, encouraging them to verify, question, criticize, and assess others arguments, engaging in constructing knowledge through various processes, and generating new knowledge through self-exploration.
- Students need to be aware that they must be an active learner taking initiatives and responsibilities in their own learning.

Shepardson (1993) asserted to the importance of cognitive engagement in making classroom effective activities that could be linked to higher order thinking skills. Continually, Anderson and Krathwohl (2001) have developed a new design for the classic Bloom's Taxonomy as a cognitive domain with a comprehensive set of definitions that appears to encompass all aspects of HOT. In other words, they created a workable and valuable way for using objectives as tools to promote students with lower-level thinking into higher-level thinking by applying the hierarchical nature of knowledge, including Remember, Understand, Apply, Analyze, Evaluate ,and Create. As a result, teachers can inquire a purposeful question in regard to revised Bloom's taxonomy, to

ensure that respondents get beyond the simple answers and can think deeper (Cochran, Conklin, and Modin, 2007). In addition, these questions could lead participants from "what" questions associated with lower-level thinking into the "how" and "why" associated with higher-level thinking (see Appendix I). Most significantly, recently, arguments have indirectly confirmed that the mathematical problem posing tasks are able to improve HOT among the verity level of students. Mathematics education experts for instance, Chin and Kayalvizhi, 2002 and Bonotto, 2008 revealed that problem posing can provide opportunities which stimulate higher order thinking by encouraging students to carry out investigations, especially open-ended situations. However, due to the limitation of mathematics courses in terms of time and subject content, these kinds of activities are inappropriate to be used on the continuing basis during the semester. Therefore, an operational problem posing tasks need to be design in order to improve HOT skills among students.

Consequently, the main objective of this communication is characterizing HOT skills that are involved in problem posing tasks from "Original textbook problem" via "structured or semi-structured problem posing situations" in "Inquiry-based-learning" environment. We assert that research findings can encourage teachers to collaborate mathematical problem posing tasks in their teaching-learning material for equipping undergraduates to HOT skills.

Mathematics Problem Posing in Inquiry-Based-Learning Paradigm

Problem posing can be defined as a generation of new problems or a reformulation of given problems in term of transformation of knowledge. Problem posing activities are considered as a powerful tool for understanding concepts , as well as , are designed based on several sources, such as, "Everyday life" problems, "Original textbook problems", "a picture or diagram" and so on. Besides, these tasks could be implemented in mathematics class through the following steps: first, students should understand the given problem posing situations (e.g., opened problem posing situation, semi-structured problem posing situation, and structured problem posing situation). Second, they are given an opportunity to discuss with their peers regarding the meaning of such problem posing situations. Third, at the end of the discussion, they are required to create their own problems respectively. Fourth, the new problems need to be solved in order to ensure whether they are solvable problems or unsolvable problems. Lastly, if the posed problems are unsolvable problems the teacher would then guide the students to alter the problems so that it would become solvable problems. Forcefully, these significant processes argue that problem posing tasks

can provide suitable condition for engaging students in specific learning process through Inquiry-Based-Learning paradigm which stress on social constructivism ideas.

The most important elements of this theory in term of problem posing approach and HOT skills are summarized in following statements (Bruner, 1966):

- Pedagogy and curriculum should require students to work together during solving problems, as a form of active learning. Therefore, Inquiry-Based-Learning reminds that learning should be based around student's questions and education should trend toward novel attitude that students' role is beyond problem solvers as well as they can become skillful in discovering and correctly generating problems. When students begin creating their own original mathematical questions and see these questions as the focus of discussion, their perception of the subject is profoundly altered. Meanwhile, these activities could embed them in high stages of revised Bloom taxonomy, namely, analyzing, evaluating and creating which are linked to HOT skills. Whereas, a suitable learning space cannot be launched, unless the teachers are familiar with their responsibility in classrooms.

- Teachers should be viewed as facilitators of learning. Due to problem posing definitions as a way to exercise real life situations, the tasks in teachers' hands is not only to facilitate learning process but to optimize it by establishing balance between conceptual understanding and procedural understanding. Furthermore, trainer's role in guiding the students during the reformation of unsolvable posed problems into solvable problems can be labeled as expeditor. On the other hand, integrating problem posing activities in mathematics lessons enable teachers to identify the level of their students' mathematical knowledge and the ways that students can lead to a better understanding of mathematics concepts. To achieve these facilities, criteria need to be made to represent and explain in term of mathematical perspectives.

- There are three types of representation of human knowledge in mathematics, namely, "Enactive", "Iconic" and "Symbolic". These notions assert to the importance of the integration of internalize knowledge as well as mastery in formal and precise mathematics' languages in order to be correctly and deeply involved in mathematical problem solving. Most important, these demonstrations are a tool for mathematics communication which consist of reviewing the part of material used for constructing a new product ,developing a novel thinking situation, thinking of the best strategy to solve tasks by

using his/her own questions that lead him/her to the solution .

In this study, researchers monitored HOT skills in Inquiry-Based-Learning environment that can undertake the mentioned elements for occurring transformation knowledge and understanding via problem posing activates.
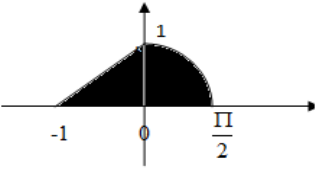
Method

Tasks were designed based on integral section which is one of the fundamental concepts in calculus. In addition, "Original Textbook Problems" were adopted from "Thomas' calculus" (2005) in regards to definite integral definition, techniques of integration, fundamental theorem of calculus and their results, application of integration in finding volume and area. Meanwhile, the level of problems were more difficult than a routine classroom activity, so that researcher can explore how problem posing tasks would engage undergraduates, as well as which types of their HOT skills would be occupied. According to problem posing and activities' framework as suggested by Abu-Elwan (2002), each of the tasks consists of two parts. Part (a) involved students in problem solving strategies, as well as Part (b) problem posing. Besides, "What..., if not?" (Brown and Walter, 2005) and "Modifying Given" (Bairac, 2005) strategies were considered as problem posing strategies that were performed through semi-structured and structured problem posing situations respectively. Semi-structured situations occur when students are asked to construct problem similar to the given problems, problems with similar situations, problems related to specific theorems, problems derived from the given pictures, real life and word problems by using knowledge, skills, concepts and relationships from their previous mathematical experiences. Besides, structured situations arise an appropriate environment for generating problems by reformulating already solved problems or by varying the conditions or questions of the given problems (Abu-Elwan, 1999; Stoyanova, 2003). Table 1 illustrates two examples of research tasks.

Continually, these tasks were implemented through test session and semi-structured interview as quantitative and qualitative approaches respectively. The time for test session was considered 90 minute and the mentioned steps in section 2 were relied on Inquiry-Based-Learning environment. Furthermore, semi-structured interview interventions were used to identify more justifications about test's outcomes. During the interview, research-teacher inquired a purposeful questions in regards to the revised Bloom's taxonomy (see Appendix I) as a way to ensure that respondents come up with beyond simple answers and were thinking more deeper (Cochran, Conklin, and Modin, 2007). Additionally, these questions were prompted when clarification is requested by participants.

Table 1: Two examples of problem posing tasks in this study

1(a)	The marginal cost of printing a poster when x posters have been printed is $\frac{dc}{dx} = \frac{1}{2\sqrt{x}}$ *dollars*. Find c (100) – c (1) namely the cost of printing posters 2-100. (Note: The marginal cost is c'(x) as well as c(x) is the total cost.) Conditions*:* Demand:
1(b)	Fist pose problem: ("Add new condition" strategy via "structured Situation") Add new condition to problem (a) New Conditions: New Demand: Solve:
1(c)	Second pose problem : ("Remove condition" strategy via "structured situation") Remove some condition of problem (a) New Conditions: New Demand: Solve:
2(a)	Find the areas of the regions enclosed by the curves and lines below $y = x^2 - 1, x = -1, x = 2, y = 0$. i. Determine problem's Condittions exactly. ii. Solve :
2(b)	Pose the problem. ("change the context" strategy via "semi- strucured situation") Create a problem related to the areas of regions for given figure i. Determine problem's Condittions exactly. and explain how evaluate new conditions. ii. Express new problem by formal mathematics connections. iii. solve:

The respondents were twenty-six first year engineering students from various faculties at Islamic Azad University of Birjand that were involved in answering the test. They were selected purposefully among moderate and high achievers who could be expected to have literacy levels sufficiently to understand questions and articulate in their posed question processes in regards to their mark in the final exam of a calculus course. Most importantly, they were first encountered in problem posing tasks. After the test session, five of them were selected for the semi-structured, task-based interview that was performed one-by-one (Roulston, 2010). Therefore, the raw data was produced by students' written

works in the test and the transcriptions of audio-taped obtained through interviews.

Qualitative content analysis was used for data analysis via inductive reasoning to condense raw data into categories or themes based on valid inference and interpretation (Zhang and Wildemuth, 2009); however, researchers generated codes based on the revised Bloom's taxonomy at the inception of data analysis. Hence, these codes were developed in terms of the comparative method.

Results and Discussions

According to findings, more than three quarters of participants (80.77%) were "Expert" or "Practitioner" in posing problems in given tasks that were performed in structured situations. These levels of performance can lead teacher-research to a range of HOT skills. According to participants' written works, specific HOT skills could be categorized to "Understanding correctly related strategies", "Interpreting the problem condition and demand in term of mathematics communication", "Creating new condition in term of formal math language", "Changing the number of conditions and generating new demand purposefully", "Analysing the constructed problem regard to solvable or unsolvable", "justifying reasons of new unsolvable problem for transferring to solvable problem". However, 73% of new conditions and demands were cosmetic, namely participants chose both "changing the values of the given data", and "changing the number of conditions" for generating problems in term of "Modifying Given" and performed them correctly, whilst only 15 percents of them preferred to use "changing the problem's question" by "What..., if not?" strategy through this situation.

These findings indicated undergraduates' abilities in procedural mathematics understanding (Marchionda, 2006) related to "techniques of integration" and "application of integration in finding volume and area". In other word, undergraduates presented a range of procedural understandings such as representing mathematical patterns, structures and regularities, using deductive arguments to justify decisions for new posed problem based on given data, and generating new problem via mathematical connections in a low-order thinking manner. Most importantly, this study founded that undergraduates had the most difficulties in using "What ...if not?" strategy for generating problems related to "characterizations of integrability concerned with continuity" and, "application of integration in finding volume a real object". These results asserted that the majority of students were unable to interpret correctly their answers , or even generalize the agreed situation part (a) via the mathematical term that it can highlight a weakness in higher- level of Bloom 's taxonomy consists of analyzing, evaluating,

and creating. As a result, two categories "Creating new condition in term of formal math language", and "Changing the number of conditions and generating new demand purposefully" occur in a routine regulation related to lower-order thinking. Hence, teacher-researchers purposefully chose 5 of the students who their written works were characterized as low-order thinking for encountering in the semi-structured interview.

The research-teacher tried to encourage participants' HOT by leading them to generate problems which are correct and different from the initial problem during the interview (see Appendix I). This method of interviewing was performed as an instructional practices that focus on sense making, self-assessment, and reflection on what worked and what needs improving in term of metacognition. Because metacognition facilitates HOT skills (Kapa, 2001) and often takes the form of internal dialogue, whilst many students remain unaware of its importance unless teachers emphasize these processes explicitly and guide students towards the reflection that needs to occur (Bransford, Brown, and Cocking, 2000). One example of this communication is as follows:

Interviwer: *Now read part b of problem 1 and say that what the problem asked you to do.*
Subject 3: *It asked to add something to the conditions of the problem... I will change the data in the problem. That is I will put 144 instead of 100 and 1 instead of 9.*
Interviwer: *Can you write your own problem in the accurate math language like the previous part?*
Subject 3: *The marginal cost of printing a poster when x posters have been printed is $\dfrac{dc}{dx} = \dfrac{1}{2\sqrt{x}}$ dollars. Find the cost of printing posters 10-144 .*
Interviwer: *You just changed the data in repetitive manner , can construct another problem with significant changing?*
Subject 3: *...ummm... Can give an example? ...*
Interviwer: *You can change the marginal cost formula to $\dfrac{dc}{dx} = \dfrac{1}{2\sqrt{x}}$*
Subject 3: *... ummm ... I would like to change it to $\dfrac{dc}{dx} = x^2 + 1$ and add an expense for operation like 1000$. Find the cost of printing posters 10-144. (HOT skills) (Time: 5min)*

On the other hand, in "What..., if not" strategy, more than half of participants (68.31%) were "Novice" or "Apprentice", this means that they had basic difficulties in problem posing process related to the altering of concrete situation such as real life tasks to a mathematical abstraction by symbolic expressions. In addition, these levels of performance can clarify a weakness in conceptual understanding. Therefore, written test works related to these tasks were unreliable for coding; however, prior studies reported that "change the context" can engage learners' thinking more than structured situations

(Abu-Elwan, 2002). Consequently, HOT skills via "change the context" strategies were sought during the interview session. The most repetitive HOT skills were found in the following categories: "Determine not given information by comparing two part (a), (b)", "Design a math graph related to a real subject", "Justify and support decisions made and conclusions reached by drawing a graph related to a mathematical theory", "manipulating information for constructing new problems in flexibility method", "Generate the problems in yours words regard to formal math language", "Create new and different problem which are solvable", "Conclude a significant pattern or structure", "Finding the differences and similarities between two part (a), (b)" and "Checking the answer by solving new posed problem". Meanwhile, research-teacher stimulates HOT skills by questions and statements which are presented in Appendix I; furthermore, when clarification was requested, she guided responds by relevant questions, writing some note and example. One example of this interview is as below:

Interviewer: *What was your difficulty in posing this problem in part b?*
Subject 3: *I can't recall the definition of defined integral by $\sum$ can you give me the formula.*
Interviewer: (write: $\int_a^b f(x)dx = \frac{b-a}{n}\sum_{k=1}^{n} f(a + \frac{k(b-a)}{n})$), *read again the question carefully and say: What are the conditions and the demands of this problem?*
Subject 3: *Make a Limit problem for the given integral.*
Interviewer: *determine the data and the information given.*
Subject 3: (*referred to note given:* $\int_a^b f(x)dx = \frac{b-a}{n}\sum_{k=1}^{n} f(a + k(b-a)n)$) *You gave the left hand part of the equality and asked us to obtain the right hand part.*

Interviewer: *So if you want to construct a problem, what should you determine?*
Subject 3: *a,b & f, and $\sum$. (HOT skill)*
Interviewer: *It means a statement in front of $\sum$.*
Subject 3: (*wrote: a=1,b=2,* $f(1 + \frac{2k}{n})$)
Interviewer: *So what happens to the Limit you need to make the problem?*
Subject 3: (*wrote* $\lim_{n \to \infty} \frac{2}{n}\sum_{k=1}^{n} f(1 + \frac{2k}{n})$))
Interviewer: *Do you have function f?*
Subject 3: *yes*
(Wrote: $\lim_{n \to \infty} \frac{2}{n}\sum_{k=1}^{n} (1 + \frac{2k}{n})^3$). *(HOT skill)*
Interviewer: *Now, constructed the problem in your words.*
Subject 3: *Shall I construct a problem for* $\lim_{n \to \infty} \frac{2}{n}\sum_{k=1}^{n} (1 + 2kn)3$ *or for* $13x3$?

Interviewer: *Suppose that you are a teacher and you want to construct a limit problem for your students to change it to the definite integral using the formula definite integral. So what will happen to you your question?*
Subject 3: *Solve the following limit using the definition of integral. (HOT skill)*
(*wrote this sentence above your formula.) (Time: 8min)*

Conclusion

Problem posing tasks demand thinking on a higher level than memorizing facts, namely the tasks require the students to understand the problem context exactly, information connected to each other, categorized, manipulated, put together in new or novel ways, and applied as new solutions to new problems, so that, they can encourage high-order - thinking among learners. Specially, problem posing tasks from "Original textbook problem" when implemented through the structured and semi-structured problem posing situations can be suggested as practical activities in mathematics classroom , whereas, they encompass subject constant and can be performed at a standard time (Saint Louis University, 2012).

In addition, the most important HOT skills via these types of tasks based on Newman (1991) can be categorized into "Interpreting the problem condition and demand in term of mathematics communication" , "Manipulating information for constructing new problems in flexibility method", "Analysing the constructed problem regard to solvable or unsolvable", "Create new and different problem which are solvable", "Conclude a significant pattern or structure", "Finding the differences and similarities between two part (a), (b)". Meanwhile, teacher, as a guide should turn pupils from low-order thinking towards high-order thinking associated with asking the "how" and "why" questions in term of the levels of the revised Bloom's taxonomy, when teachers apply these questions in a continual manner, self-questioning as a metacognition skill can gradually improve in learners which will be conducive to promoting their HOT skills (Kapa, 2001). For instance, "How would you construct a new condition/demand?", "How do you justify the logic of the answer?", "What changes would you make to solve new posed problem?" and "Can you generate the problems with your words based on the formal math language?". However, based on the results of this study, it is recommended that researchers investigate the effects of mathematical problem posing activities mentioned on undergraduates' higher order thinking skills in terms of a survey with quantity approach. Ramirez and Ganaden (2008) pointed out that, despite their assumption, the chemistry tasks with creative activities is not significantly different from the instructions with no creative activities to improve the higher order thinking skills of students.

Consequently, we expect this study to be able to encourage teachers to move toward novel perspectives of classroom activities by collaborating problem posing and high-order thinking approaches through face to face classroom interactions.

References

Abu-Elwan, R. (1999). The Development of Mathematical Problem Posing Skills For Prospective Teachers. In A. Rogerson (Ed.), *Proceedings of the International Conference*

on *Mathematics Education into 21st Century. Social challenges, Issues, and Approaches* (pp. 1-8). Cairo, Egypt.

Abu-Elwan, R. (2002). *Effectiveness of Problem Posing Strategies on Prospective Mathematics Teachers' Problem Solving Performance*. Sultan Qaboos University.

Anderson, L. W., and Krathwohl, D. R. (Eds.) (2001). *A taxonomy for learning, teaching, and assessing: A revision of Bloom's taxonomy of educational objectives*. New York: Longman.

Bairac, R. (2005). Some Methods for Composing Mathematical Problems. *International Journal for Mathematics Teaching and Learning*, Available online at: http://www.cimt.plymouth.ac.uk/journal/bairac.pdf.

Bigge, M. L. (1976). *Learning Theories for Teachers (3rd ed.)*. New York, NY: Harper & Row.

Bransford, J. D., Brown, A., and Cocking, R. R. (2000). *How People Learn: Brain, Mind, Experience and School*. Washington, D.C.: National Academy Press.

Bonotto, C. (2008). *Realistic Mathematical Modeling and Problem Posing*. University of Padova.

Brown, S., and Walter, M. I. (2005). *The Art of Problem Posing (3rd ed.)*. Mahwah, NJ: Lawrence Erlbaum Association.

Bruner, J. S. (1966). *The Process of Education*. Cambridge, MA: Harvard University Press.

Chin, C., and Kayalvizhi, G. (2002). Posing Problems for Open Investigations: What Questions Do Pupils Ask?, *Research in Science & Technological Education*, 20(2), 1-18.

Cochran, D., Conklin, J., and Modin, S. (2007) . A New Bloom: Transforming Learning. *Learning and Leading with Technology. 34*(5):22–25.

Engelbrecht, J., Bergsten , C., and Kagesten, O. (2009) . Undergraduate Students' Preference for Procedural to Conceptual Solutions to Mathematical Problem. *International Journal of Mathematical Education in Science and Technology*. 40(7): 927-940.

Kapa, E. (2001). A Metacognitive Support during the Process of Problem Solving In a Computerized Environment. *Educational Studies in Mathematics , 47*, 317-336.

Marchionda, H. (2006). *Procedural and Conceptual Understanding of Fractions and the Effects of Inquiry-based Learning.* Clemson University.

Newman, F. M. (1991). Promoting Higher Order Thinking in Social Studies: Overview of A Study of 16 High School Departments*. Theory and Research in Social Education*.19: 324-340.

Ramirez, R. P. B., and Ganaden, M. S. (2008). Creative Activities and Students' Higher Order Thinking Skills. *Education Quarterly.* U. P. College of Education. 66(1): 21-33.

Roulston, K. (2010). *Reflective Interview: A Guide to Theory and Practice*. London: Sage.

Saint Louis University. (2012). Success in Mathematic Tips. *Academic Tips. Org.* 20 July 2012, Retrieved from http://www.academictips.org/acad/mathematics/ successinmathematics.html .

Shepardson, D. P. (1993). Publisher-based Science Activities of the 1980s and Thinking Skills. *School Science and Mathematics. 93*(5): 264-268.

Shu Mei, Y., and Yan, Z. (2005). Higher-Order Thinking in Singapore Mathematics Classrooms. *Centre for Research in Pedagogy and Practice: National Institute of Education,* Singapore.

Stoyanova, E. (2003). Extending Students Understanding of Mathematics via Problem Posing. *Australian Mathematics Teacher*, 59(2), 32–40.

Thomas, A., Glenda, T., and Small, B. (2000). Higher Order Thinking–It's HOT! *Centre for Development and Learning*. Louisiana. 1(3).

Thomas, G. B. , Weir, M. D., Hass, J., and Giordano , F. R. (2005). *Thomas' Calculus*. United States of America: Pearson Education ,Inc.

Weiss, E. (2003). Problem-Based Learning in The Information Age: Designing Problems to Promote Higher-order Thinking. *New Direction for Teaching and Learning, Wiley Periodicals, Inc.* 95:25-31.

Zhang, Y., and Wildemuth, B. M. (2009). Qualitative Analysis of Content. In B. Wildemuth (Ed.), *Applications of Social Research Methods to Questions in Information and Library Science*. Westport, CT: Libraries Unlimited, 308-319.

APPENDIX I: Categories of semi-structured interview questions based on cognitive process dimension

Cognitive Process Dimension	Questions/Directives
Remember	Can you recall "definite integral definition" ,"techniques of integration"," fundamental theorem of calculus and their results"," application of integration in finding volume and area" ....?
Understand	What are the conditions of this section of problem? What are the demands of this section of problem? Can you determine the data and the information need to be found?
Apply	Can you plan new problems? Know exactly what to do next. If you are unsurely, do whatever that seems logical through reasoning. Determine the sub-goal(s). Improve the plan (another way). Need to arrange the information Can you carry out the problem posing strategy? How would you construct a new condition by "changing the context" strategies for posing a new problem? How would you construct a new demand by "changing the context" strategies for posing a new problem? What is new condition that you would add or remove to previous condition? What is new condition that you would add or remove to previous condition? How would you evaluate your question / Can solve new problem?
Analyze	How would you analyze the situation in mathematical terms, and extend prior knowledge presented in part (a)? Can you identify that new problem is solvable or unsolvable? What is the meaning of the answer? How do you justify the logic of the answer? Put in the units to understand the meaning. Conclude a significant pattern or structure. If you further reading, can find some another clues? Compare given data part (a) and (b)
Evaluate	How would you justify and support decisions made and conclusions reached by drawing a graph related to a mathematical theory? Simply look back again (recap). Checking the logic of the equation arrangement. Checking the answer by interpreting. Reading to see if the goal is achieved as required by the question. Checking the plan/analysis. Checking the steps, go back and do again. Another way of calculation to check.
Create	Can you design a math graph related to a real subject? Can you generate the problems in yours words regard to formal math language? What changes would you make to solvable new posed problem? Can you construct more than 1 new problem?